ε, porosity; λ, root of the characteristic equation; μ_0, gas viscosity; ν, dimensionless frequency; ξ, dimensionless coordinate; ρ, volume particle concentration; ω, frequency; γ, $\varkappa$, σ, parameters introduced into (18). Indices: 0, undisturbed state of the bed; ', pulsations of hydrodynamic quantities and differentiation of α with respect to ρ; *, state under the gas-distributing grid.

LITERATURE CITED

1. R. Jackson, "The mechanics of fluidized beds," Trans. Inst. Chem. Eng., Part I, 41, 13-20 (1963).
2. R. L. Pigford and T. Baron, "Hydrodynamic stability of a fluidized bed," Ind. Eng. Chem. Fund., 4, 81-87 (1965).
3. J. D. Murray, "On the mathematics of fluidization," Part I, J. Fluid Mech., 21, 465-476 (1965).
4. T. B. Anderson and R. Jackson, "Fluid mechanical description of fluidized beds. Stability of a state of uniform fluidization," Ind. Eng. Chem. Fund., 7, 12-21 (1968).
5. Yu. A. Buyevich, "Statistical hydromechanics of disperse systems," J. Fluid Mech., 56, 313-336 (1972).
6. M. É. Aérov and O. M. Todes, Hydraulic and Internal Principles of the Operation of Equipment with Stationary and Fluidized Beds [in Russian], Khimiya, Leningrad (1968).
7. R. Jackson, "Theoretical mechanics of fluidization," in: Fluidization [Russian translation], Khimiya, Moscow (1974), pp. 74-121.
8. T. B. Anderson and R. Jackson, "A fluid mechanical description of fluidized beds. Comparison of the theory and experiments," Ind. Eng. Chem. Fund., 8, 137-144 (1969).
9. J. Verloop and P. M. Heertjes, "Periodic pressure fluctuations in fluidized beds," Chem. Eng. Sci., 29, 1035-1042 (1974).
10. A. I. Tamarin and G. I. Kovenskii, "Propagation of static-pressure oscillations in a fluidized bed," Teor. Osn. Khim. Tekhnol., 6, 83 (1972).
11. V. P. Golo and V. P. Myasnimov, "Dispersion phenomena in a fluidized bed," Prikl. Mat. Mekh., 39, 747 (1975).
12. Yu. A. Buyevich, "Kontinuumsmechanik konzentrierter Suspensionen," Z. Angew. Math. Mech., 56, 379-388 (1976).
13. Yu. A. Buyevich and I. N. Shchelchkova, "Flow of dense suspensions," Progr. Aerospace Sci., 18, 121-150 (1978).
14. R. I. Nigmatullin, Principles of the Mechanics of Heterogenous Media [in Russian], Nauka, Moscow (1978).
15. Yu. A. Buevich and I. N. Shchelchkova, "Rheological properties of homogeneous finely divided suspensions: steady-state flows," Inzh.-Fiz. Zh., 33, 872-879 (1977).
16. B. S. Endler, "The effective viscosity and thermal conductivity of a dispersed medium," Inzh. Fiz. Zh., 39, 110 (1979).
17. J. O. Hinze, "Turbulent fluid and particle interaction," in: Progress in Heat and Mass Transfer, Vol. 6, Pergamon Press, New York–London (1972), pp. 433-452.

METHOD OF INTERPOLATING DATA IN DETERMINING THE RHEOLOGICAL PARAMETERS OF A LIQUID

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The least-squares method is applied to determine the parameters in the rheological equation of state for the liquid over wide ranges in the velocity gradient and tangential stress.

The rheological parameters of liquids have major effects on the hydromechanical, thermal, and mass-transfer processes [1], and therefore correct determination of the rheological equation of state for a liquid is a basic problem in rheology [2, 3].

The ranges in strain rate and stress for a given object frequently constitute 4-6 orders of magnitude, so mathematical description of experimental values usually involves piecewise approximation for individual ranges in

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the input and output parameters by means of a classical least-squares method [4, 5]. This complicates calculations related to the logical selection of an equation in relation to the range in the tangential stress or velocity gradient. There are major difficulties in describing the entire ranges in the input and output parameters by means of a single rheological equation of state. We can demonstrate this on the determination of the rheological equation of state for glycerol, which is one of the simplest Newtonian liquids:

$$\tau = \mu\dot{\gamma}.$$

Table 1 gives results from the capillary viscometry of glycerol at t = 60°C.

The tangential stresses and velocity gradient were determined as functions of the pressure gradient and the corresponding flow rate from the following formulas [3]:

$$\tau_{\mathrm{w}} = \frac{\Delta p}{4l} d_{\mathrm{c}}; \ \dot{\gamma}_{\mathrm{w}} = \frac{32q}{\pi d_{\mathrm{c}}^3}.$$

The viscometry results were backed up by processing the dwell-time distribution via the following equations [6]:

$$\dot{\gamma} = -\frac{4l}{d_{\mathrm{c}} C_{\mathrm{i}} t^3} \left(\int_{t_{\mathrm{d}}}^{t_0} C_{\mathrm{i}} t dt \right)^{0.5} \left(\int_{t_{\mathrm{d}}}^{t} C_{\mathrm{i}} t dt \right)^{0.5};$$

$$\tau = \frac{\Delta p d_{\mathrm{c}}}{4l} \left(\frac{\int_{t_{\mathrm{d}}}^{t} C_{\mathrm{i}} t dt}{\int_{t_{\mathrm{d}}}^{t_0} C_{\mathrm{i}} t dt} \right)^{0.5}.$$

The upper part of Table 1 shows that the results from the classical least-squares method for the entire ranges of input and output quantities at 60°C are such that glycerol behaves as a viscoplastic liquid with a limiting shear stress $\Theta = 0.055$ N/m^2.

The coefficients in the rheological equation of state obtained by this method

$$(\tau \cdot 10^3) = 55 + 104.9\dot{\gamma}$$

were examined on Student's t-test, which demonstrated significance for both coefficient [7]. This conclusion that glycerol is a viscoplastic liquid conflicts with the evidence that it is a Newtonian liquid. Also, there are impermissibly large relative deviations between the theoretical and experimental values for the tangential stresses at low velocity gradients, which indicates either an error in the experiment or that the entire range of the input and output parameters cannot be fitted to a single equation by least squares.

In our view, this occurs because the criterion for minimizing the sum of the squares of the deviation in the least-squares method

$$S_1^2 = \sum_{i=1}^{n} [(b + ax_i) - y_i]^2\} \min \tag{1}$$

gives good results when the experimental data relate to narrow ranges in the input and output parameters.

If the ranges in the input and output parameters are wide, the least-squares approximation gives an unsatisfactory result because the theoretical and observed values of the function come together in absolute magnitude in the region of their largest numerical values at one end of the range of measurement, where (1) shows that one obtains the least squares of the deviations. On the other hand, at the other end of the range of the measurements, namely at small numerical values for the function [where the deviations in absolute magnitude are small and make the least contribution to the sum in (1)], the relative deviations between the theoretical and experimental values may be hundreds or thousands of percent (Table 1).

Therefore, to approximate the data over wide ranges in the parameters it is desirable for condition (1) to be replaced by a condition in which one minimizes the sum of the relative squares of the deviations:

TABLE 1. Experimental Data and Theoretical Results on the Dependence of Tangential Stress on Velocity Gradient

$\dot{\gamma}$	0,01	0,1	2	10	100	1000	Experimental values
$\tau\cdot 10^3$	1	12	180	900	11000	105000	
$\tau\cdot 10^3$	56	65,5	265	1105	10555	105056	LSF
ε	5500	440	47	22,7	—4,5	0,5	$S_1^2=0{,}25631\cdot 10^6$
							$S_2^2=0{,}30452\cdot 10^4$
$\tau\cdot 10^3$	1,013	10,024	200,25	1001,2	10012	100120	RLSF
ε	1,3	—16	11	11	—9	—4,5	$S_1^2=0{,}248\cdot 10^8$
							$S_2^2=0{,}6278\cdot 10^{-1}$

$$S_2^2=\sum_{i=1}^{n}\left(1-\frac{b+ax_i}{y_i}\right)^2\Bigg\}\min. \tag{2}$$

The normal equations here are

$$\sum_{i=1}^{n}\left(\frac{1}{y_i}\right)-b\sum_{i=1}^{n}\left(\frac{1}{y_i}\right)^2-a\sum_{i=1}^{n}\left(\frac{x_i}{y_i^2}\right)=0 \tag{3}$$

and

$$\sum_{i=1}^{n}\left(\frac{x_i}{y_i}\right)-b\sum_{i=1}^{n}\left(\frac{x_i}{y_i^2}\right)-a\sum_{i=1}^{n}\left(\frac{x_i}{y_i}\right)^2=0,$$

from which we have formulas for *a* and b:

$$a=\frac{\sum\limits_{i=1}^{n}\left(\frac{1}{y_i}\right)^2\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i}\right)-\sum\limits_{i=1}^{n}\left(\frac{1}{y_i}\right)\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i^2}\right)}{\sum\limits_{i=1}^{n}\left(\frac{1}{y_i}\right)^2\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i}\right)^2-\left[\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i^2}\right)\right]^2};$$

$$b=\frac{\sum\limits_{i=1}^{n}\left(\frac{1}{y_i}\right)\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i}\right)^2-\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i}\right)\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i^2}\right)}{\sum\limits_{i=1}^{n}\left(\frac{1}{y_i}\right)^2\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i}\right)^2-\left[\sum\limits_{i=1}^{n}\left(\frac{x_i}{y_i^2}\right)\right]^2}. \tag{4}$$

The lower part of Table 1 gives results from (4) for the tabulated values of the tangential stress in relation to velocity gradient, while the approximating rheological equation of state is

$$(\tau\cdot 10^3)=-0.0119+100{,}12\dot{\gamma},$$

where $\Theta=1.19\cdot 10^{-5}$ N/m^2 and is not significant on Student's t-test.

The final rheological equation of state for glycerol at 60°C is

$$\tau\cong 0.1\dot{\gamma},$$

which corresponds to the results of [8].

In a similar way one can determine the equations of state and the numerical values of the parameters for an extensive class of non-Newtonian liquids with the experimental data approximated over wide ranges in the tangential stresses and velocity gradient.

NOTATION

a, b, parameters of linear regression equation; C, stay period distribution function; d, diameter, m; n, number of experiments; *l*, length, m; i, experiment number; q, volume flow rate, m^3/sec; t, time, sec; S_1^2, S_2^2, sums of squares of absolute and relative deviations; x, y, argument and function; $\dot{\gamma}$, velocity gradient, sec^{-1}; Δ, measured difference; ε, relative deviation, %; Θ, limiting shear stress, N/m^2; μ, viscosity, nsec/m^2; Σ, sum; τ, shearing stress, N/m^2. Indices: d, delay; i, indicator; c, capillary; o, end; w, wall.

LITERATURE CITED

1. N. V. Tyabin, Rheological Cybernetics [in Russian], Volgograd. Pravda, Volgograd (1978).
2. F. R. Eirich (ed.), Rheology. Theory and Applications [Russian translation], IL, Moscow (1962).
3. M. Reiner, Deformation, Strain, and Flow: An Elementary Introduction to Rheology [Russian translation], Gostoptekhizdat, Moscow (1963); Lectures on Theoretical Rheology [Russian translation], Nauka, Moscow (1965).
4. N. V. Tyabin, "Theory of the anomalous viscosity in dispersed systems," Zh. Tekh. Fiz., 25, No. 2, 339 (1955).
5. P. F. Ovchinnikov, N. N. Kruglitskii, and N. V. Mikhailov, Rheology of Thixotropic Systems [in Russian], Naukova Dumka, Kiev (1972).
6. A. B. Golovanchikov and N. V. Tyabin, "Determination of the rheological parameters of the grid from the response functions of flow structures in capillaries," Inzh.-Fiz. Zh., 40, No. 4, 651 (1981).
7. V. E. Gmurman, The Theory of Probability and Mathematical Statistics [in Russian], Vysshaya Shkola, Moscow (1977).
8. K. P. Mishchenko and A. A. Ravdel' (ed.), A Short Handbook of Physiochemical Quantities [in Russian], Khimiya, Leningrad (1967).

FLOW RETARDATION DURING DEFORMATION IN VARIOUS MODES

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Retardation of the flow of a polymer fluid during elongation in various modes is analyzed on the basis of theory and experiments.

It has been discovered in earlier studies [1, 2] that a polymer fluid (polyethylene melt) under tension with a constant deformation rate $\varkappa$ can exhibit two ranges of deformation as a solid almost (with essentially elastic strains developing). The usually observed first range is found during the beginning of the deformation process and extends over periods shorter than the relaxation time. The second loss of fluidity can already occur after the flow has developed. The tension force F(t), varying in time, was found to pass through two maxima. Both ranges of suppressed flow corresponded to an increase of force F. As the deformation rate $\varkappa$ was increased, both ranges eventually merged into one. An appraisal of the recurring flow based on the second increasing of the force suggests that this property is characteristic of polymers which crystallize (polyethylene) as well as of those which do not (polystyrene).

After the recurring solidification there either again develops a flow (low-density polyethylene at 125°C) or occurs a rupture of the elongated specimen (high-density polyethylene at 130°C). Thorough data on rupture of monodisperse polymer fluids can be found in another report [3], where it has been attributed to a loss of fluidity. It remains unclear, however, whether or not solidification had recurred in that study [3].

The loss of fluidity is, it seems, manifested differently in different polymer fluids. Thus, e.g., no loss of fluidity (no range of constant irreversible strain) was found in grade P-20 polyisobutylene elongated at 44°C within the same range of strain and strain rates as polyethylene had been earlier [1]. Here the force F(t), varying in time, passed through only one maximum and continued to decrease monotonically.

It is to be noted that polyisobutylene at 44°C and low-density polyethylene at 125°C have approximately the same viscosity, $\eta \sim 3 \cdot 10^5$ Pa·sec and modulus of high-elasticity $G_e \sim 10^3$ Pa. Their flow curves within the given range of shearing strain rates ($\varkappa < 1$ sec^{-1}) also do not differ much.

According to another study [4], a polymer fluid in the range of nonlinear deformation often cannot be described by the viscosities and the relaxation times alone. Describing the loss of fluidity requires still another parameter β^* ($0 \leq \beta \leq 1$), which characterizes the aptitude of the material for strain orientation. The

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